Fourier Analysis [6 CFU] (first semester)

Outline: The denomination *Fourier Analysis* refers to the Fourier series, the Fourier and Laplace transforms, and other classical integral transforms, including the more recently developed *time-frequency analysis*. These notions play a fundamental role in both classical and modern analysis, in particular in the study of ordinary and partial differential equations. This theory also provides the language for some branches of engineering, and has important applications in many sciences: physics, geology, signal theory, cardiology and so on.

Prerequisites (in parenthesis the corresponding courses in Trento):

Differential and Integral Calculus, with Fourier series and ODEs *(Analisi I, II).*
Measure theory and Lebesgue integration, L^p-spaces *(Analisi II).*
Linear algebra *(Geometria I).*
General topology *(Geometria II).*
Elements Banach and Hilbert spaces *(Analisi Funzionale).*

Main contents: (the detail is provided by the lecture notes)

Review of Fourier series. Trigonometric and exponential representation of Fourier series of periodic functions. Pointwise and L^2-convergence of Fourier series. Solution of PDEs by variable separation and Fourier series. Fourier series and musical theory.

Schwartz distributions. Test functions. Distributions. Differentiation of distributions. Support and order of distributions. Compactly supported distributions. Tempered distributions. Convolution.

Fourier transform. The Fourier transform in L^1, in $\mathcal{S}$, in $\mathcal{S}'$, in L^2. Main properties. Fourier transform and PDEs.

Laplace transform. Laplace transform of functions and distributions. Main properties. Laplace transform and ODEs.

The formulation of examples and the solution of problems are an important part of the study of Fourier analysis. Several exercises will be provided and solved during the classes.

Texts: Teacher's lecture notes (available on the web).

Complementary texts (all available at the University Library in Povo):

C. Gasquet, P. Witomski: *Fourier analysis and applications.* Springer, New York 1999
G. Gilardi: *Analisi tre.* McGraw-Hill, Milano 1994
M. Marini: *Metodi matematici per lo studio delle reti elettriche.* C.E.D.A.M., Padova, 1999

Modality of exam: Written and oral examinations.

Schwartz distributions, Fourier and Laplace transforms, and their use in the analysis of differential equations. Linear systems. Time-frequency analysis.